

# Critical State Modeling of Lower-bound threshold Stress for Clay Subgrade

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**ABSTRACT:** Railway track foundation with fine-grained subgrade, under loading of cyclic nature, can build up excess pore pressure and results in progressive shear failure at a stress level less than its static undrained strength. It is widely accepted a threshold stress level exists, where loading above such level causes large deformation; otherwise stability of vertical track profile will be assured. This paper described the theoretical model that predicts the threshold stress of clay soil. The threshold stress so determined represents the lower-bound solution for bearing strength capacity of clay subgrade which normally exists in an unsaturated state. Developed from the “original Cam-clay” model and validated by series of undrained cyclic triaxial test data on reconstituted kaolin clay, the proposed theoretical model enables the prediction of threshold stress to be made from the fundamental properties of the soil, based on current state of stress and stress history of the subgrade soil. The model reserved its ability to further expand into predicting the threshold stress of soil in its unsaturated state.

**KEYWORDS:** Cyclic Loading; Constitutive modeling; Clay; Threshold Stress

## 1 INTRODUCTION

With the passing of train wheels, subgrade soil beneath railway tracks experiences stress of cyclic nature that can result in progressive shear failure at a stress level lower than its monotonic strength. There exists a threshold stress in clay subgrade. When the track subgrade will to be stressed below the threshold stress level, stabilization of the track vertical alignment can be achieved.

Clay subgrade normally exists in unsaturated state can become highly saturated in wet season. The dissipation of the excess pore water pressure built up in the subgrade will be a slow process. Test simulation using undrained triaxial tests on a saturated soil specimen to determine the threshold stress will produce a “lower-bound” solution for subgrade bearing strength in the design of railway track foundation with unsaturated clay subgrade.

Undrained cyclic tests conducted on reconstituted saturated kaolin clay (normal-consolidated at 300 kPa) showed that, under the continuous application of cyclic deviator stress ( $q$ ) of sinusoidal shape, with cyclic loading frequency of 1Hz simulating the passing axles of a train travelling at a speed of 50 km/hour, excess pore water pressure gradually built

up over each cycle of deviator stress application. As cyclic deviator stress oscillates in a sinusoidal way between the maximum and the near zero values, the excess pore water pressure gradually builds up, oscillating in the same phase as the deviator stress. (Fig 1a) shows the maximum excess pore water pressure corresponding to the peak deviator stress of each loading cycle. On the  $p'-q$  plot (Figure 1b), cyclic deviator stress smaller than threshold stress produces peak stress path which represents the path of most critical and vulnerable effective stress state of each cycle. The peak stress path migrated leftward and become stagnant before the critical state line (CSL) as the cyclic stress path of each cycle formed hysteresis loop, signifying the arrival of stress equilibrium state (Sangrey 1969). Concurrently, the axial strain developed and subsequently become stabilized. The threshold stress (Ansal 1989; Heath et al. 1972; Lefebvre 1986; Procter 1984; Sangrey 1969; Waters 1968) (or critical level of cyclic stress) will be the maximum possible cyclic deviator stress ( $q$ ) (at point “Ts”) that arrives at the line of stress equilibrium state (LCSES), beyond which, soil yields at large strain.

There have been constant attempts and advances by researchers to model the cyclic behavior of clayey soil (Carter et al. 1982; Egan and Sangrey 1978). This paper began by describing the existence of cy-

clic stress equilibrium state surface (CSESS) and its theoretical model, from which prediction equations for the lower-bound threshold stress was derived. The critical state model described here was validated by test results from series of undrained cyclic triaxial tests conducted on reconstituted kaolinite clay, with stress history ranging from normal-consolidated state to over-consolidation ratio of 20.

The proposed model can be further expanded for predicting threshold stress of soil in its unsaturated state, by taking into account the potential increase in threshold stress due to densification under cyclic compression. In this regard, a multiplier equation which provides a correlation between the Degree of saturation ( $S$ ), the pore pressure coefficient ( $B$ -value) and the load-time duration will have to be established.

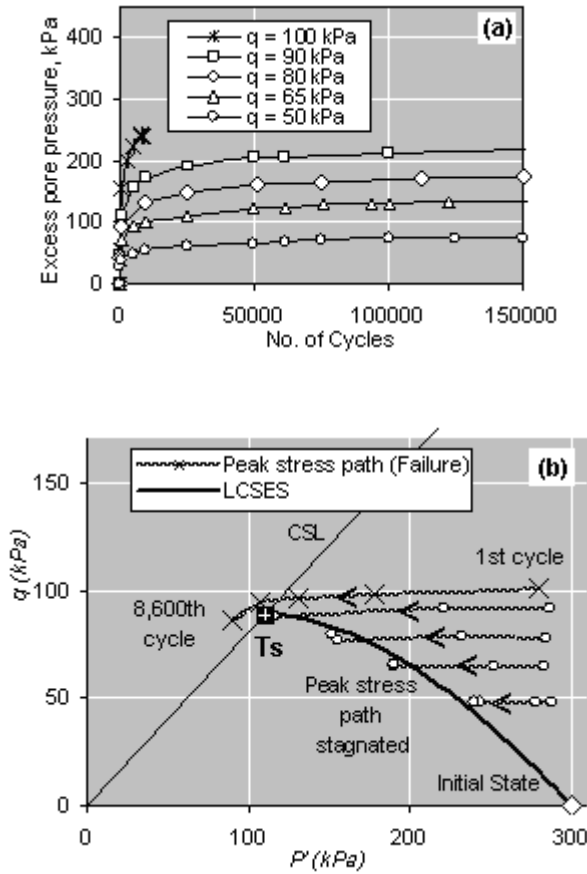


Figure 1 Result of cyclic undrained compression conducted on kaolinite clay (normal-consolidated at 300 kPa): (a) Generation of excess pore water pressure correspond to the peak stress of each cycle; (b) Peak stress path of each cyclic deviator stress ( $q$ ) smaller than the threshold stress stagnated near the line of cyclic stress equilibrium state (LCSES).

## 2 RELEVANT NOTION(S) OF “ORIGINAL CAM-CLAY”

The rational development of original Cam-clay model (Schofield and Wroth 1968) and subsequently, modified Cam-clay model (Wood 1990) demonstrated a means by which various simple mechanical models that represent the laboratory behavior of remolded soil can be explained. The general stress invariants and the relevant notion(s) of “Original Cam-clay” are summarized below:

$$\text{Mean effective stress, } p' = \frac{1}{3}(\sigma'_1 + 2\sigma'_3) \quad (1)$$

$$\text{Axial deviator stress, } q = \sigma_{dev} = \sigma'_1 - \sigma'_3 \quad (2)$$

where  $\sigma'_1, \sigma'_2, \sigma'_3$  are the principal effective stresses.

In general, the stress state of soil can be at any one time represented by a point in the 3-D space represented by the co-ordinates parameter  $p'$ ,  $q$  and  $v$ , with  $v$  being the specific volume of soil,

$$v = 1 + e \quad (3)$$

where  $e$  is the void ratio of soil.

The original Cam-clay model required the following four parameters to be specified, which are fundamental properties that can be obtained from a standard static triaxial test. They are:

- $\lambda$  Normal consolidation volumetric compression index – the gradient of the normal consolidation line in  $v - \ln p'$  plane
- $\kappa$  Swelling/Recompression index – the average gradient of the swelling and recompression line in  $v - \ln p'$  plane.
- $\Gamma$  Specific volume of soil at  $p'=1$  kPa on the critical state line CSL.
- $M$  The value of the stress ratio  $q/p'$  at the critical state condition, which defines the gradient of CSL

The current yield function of the soil represents the volumetric yield locus (yield curve) on the  $p' - q$  plane with its isotropic state base line ( $q = 0$ ) coincides with a specific swelling/recompression line defined by its normal pre-consolidation pressure  $p'_o$  (Figure 2). Yielding of the soil occurs whenever the stress state of soil satisfies the criterion as follows:

$$\frac{q}{Mp'} + \ln\left(\frac{p'}{p_{cr}'}\right) = 1 \quad (4)$$

where  $p_{cr}'$  is the mean effective stress at the intersection of the CSL and the yield curve.

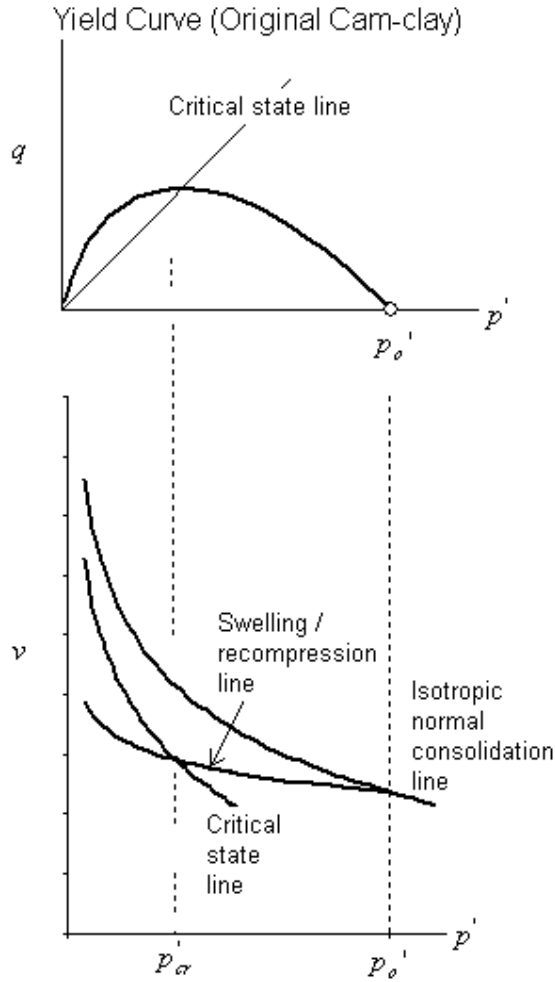


Figure 2 Yield curve and compression plane for original Cam-clay

### 3 AN UNDRAINED CYCLIC COMPRESSION MODEL

Developed from the original Cam-clay model, the proposed model described the functional equation which defines the cyclic stress equilibrium state surface (CSESS) for clayey soil under undrained cyclic compression. The importance of CSESS as it influences the cyclic stress responses for different consolidation state was illustrated, following which the theoretical model for predicting the threshold stress can be derived.

#### 3.1 Cyclic Stress Equilibrium State

The basic idea of volumetric compatibility during undrained compression suggests that a change in the soil volume ( $\Delta v$ ) can involve plastic volumetric change ( $\Delta v_p$ ) and elastic volumetric change ( $\Delta v_e$ ). In general, total volume change ( $\Delta v$ ) will be

$$\Delta v = \Delta v_p + \Delta v_e \quad (5)$$

For undrained triaxial compression,  $\Delta v = 0$ ,

$$\Delta v_p = -\Delta v_e \quad (6)$$

For a clay that has been normally-consolidated to a pre-consolidation pressure  $p'_o$ , the size of current yield function represents the boundary of the stress state ( $p', q$ ) in the “ $p'-q-v$ ”space, within which there can only be elastic swelling ( $\Delta v_e$ )<sub>swelling</sub> occurs prior to any possible elastic contraction ( $\Delta v_e$ )<sub>contraction</sub> (Figure 3).

Under undrained cyclic compression with cyclic deviator stress ( $q_{cyc}$ ), the remolding of clay causes an apparent over-consolidation for a normally-consolidated clay (Lefebvre 1986). From equation (6),

$$(\Delta v_p)_{contraction} = (\Delta v_e)_{swelling} \quad (7)$$

Since plastic volumetric change, i.e. plastic contraction ( $\Delta v_p$ )<sub>contraction</sub> appeared to be a non-reversible process in an undrained compression, Equation (7) implies there will be no elastic contraction ( $\Delta v_e$ )<sub>contraction</sub> during the unloading process. The process of plastic contraction could, in general, continues until it equals the total elastic swelling  $\Sigma(\Delta v_e)_{swelling}$  that can possibly occur within the original yield function (Figure 3), such that,

$$\begin{aligned} \Sigma(\Delta v_p)_{contraction} &= \Sigma(\Delta v_e)_{swelling} \\ &= function \left\{ \lambda, \kappa, (p', q_{cyc})_{original\ yield\ function} \right\} \end{aligned} \quad (8)$$

Therefore, the boundary of cyclic stress equilibrium state on the  $p'-q$  plane (at constant volume) will have a function identical to the original yield function as follows:

$$q = Mp' \ln \left( \frac{p'_o}{p'} \right) \quad (9)$$

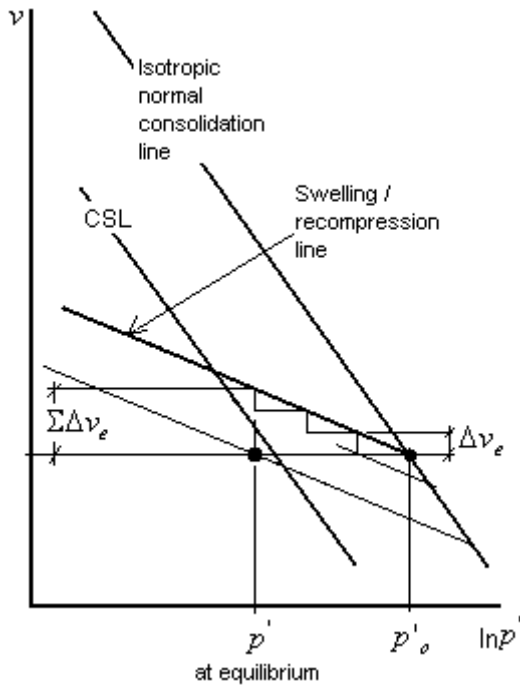


Figure 3 Volumetric compressibility of soil

It is reasonable to postulate that cyclic stress equilibrium state boundary exist for each pre-consolidation pressure along the isotropic normal consolidation line (incl), forming a cyclic stress equilibrium state surface CSESS within the stress state boundary surface SSBS. Figure 4 showed the CSESS in a  $p' - q$  plot normalized with the equivalent consolidation pressure  $p'_e$ .

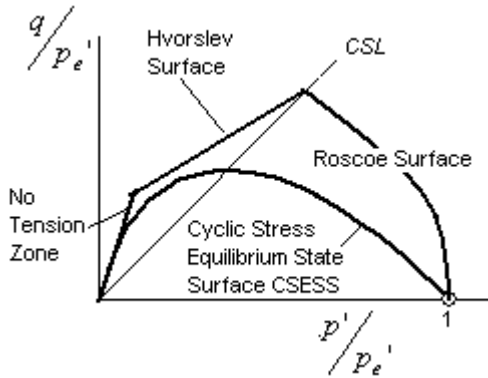


Figure 4 The existence of cyclic stress equilibrium state surface CSESS within the overall stress state boundary surface SSBS.

### 3.2 Significance of Cyclic Stress Equilibrium State Surface

Under undrained cyclic compression, the state of cyclic stress path relative to the cyclic stress equilibrium state surface determines the specific stress response as follows:

#### 3.2.1 Cyclic Stress State within CSESS

In the case of the state of cyclic stress path originates and moves within the CSESS,

$$\Sigma(\Delta v_e)_{\text{swelling}} = \Sigma(\Delta v_e)_{\text{contraction}} = 0 \quad (10)$$

The cyclic stress responds in an elastic manner, and stress path forms hysteresis loop. The state of stress equilibrium is said to be inherent when the cyclic stress path occur within the CSESS.

#### 3.2.2 Cyclic Stress State beyond CSESS

For cyclic compression that goes beyond CSESS, cyclic stress response, depends largely on the stress history of soil, i.e. over-consolidation ratio, are as follows:

- Normally and Lightly Over-consolidated Soil

For soil exists in contractive state, peak stress path moves horizontally to the left (as shown in Figure 1b). Two distinctive patterns can possibly occur depending on the level of cyclic deviator. One in which the peak stress path first reaches the critical state line CSL where large soil deformation follows (as in path (a) in Figure 5). The other, the peak stress path reaches the cyclic stress equilibrium state surface where stress hysteresis loop will be formed (path (b) in Figure 5). The threshold stress  $q_t$  (or critical level of cyclic loading) will be the maximum possible cyclic deviator stress that will arrive at the cyclic stress equilibrium surface, beyond which the peak stress path will approach the critical state line (CSL) and soil deform with large strain.

- Heavily Over-consolidated Soil

The tendency of over-consolidated clay to dilate means that the plastic swelling  $(\Delta v_p)_{\text{swelling}}$  of soil will have to be balanced by the elastic contraction  $(\Delta v_e)_{\text{contraction}}$  at constant volume.

However, such tendency violates the basic internal mechanism for volumetric change as in Equation (7), causing instability within soil mass when cyclic compression become unsustainable as large deformation occurs.

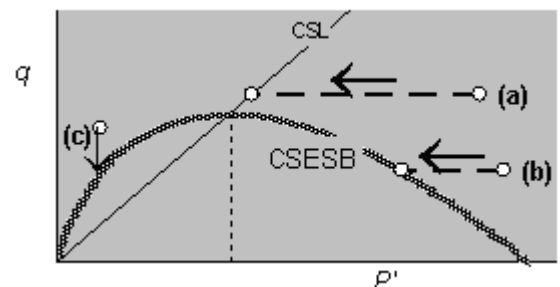


Figure 5 The movement of peak stress path in relation to CSESB

### 3.3 Prediction of Threshold Stress

In view of cyclic stress path and peak stress path response in relation to the CSESS typical of stress history, theoretical prediction model for the threshold stress of clay can be derived from the general equations (8) and (9) and threshold stress of clay can be established from the general solution as shown below:

In the case of normally and lightly over-consolidated soil, when

$$\frac{p_i'}{p_o'} \geq e^{\left(\frac{\lambda}{\kappa-\lambda}\right)} \quad (11)$$

Threshold stress ( $q_t$ ),

$$q_t = \frac{M}{e} p_o'^{\left(1-\frac{\kappa}{\lambda}\right)} p_i'^{\left(\frac{\kappa}{\lambda}\right)} \quad (12)$$

where  $p_i'$  is initial effective mean stress (or current initial state) of soil, and  $p_o'$  is the past maximum consolidation pressure (or pre-consolidation pressure).

For heavily consolidated clay, when

$$\frac{p_i'}{p_o'} \leq e^{\left(\frac{\lambda}{\kappa-\lambda}\right)} \quad (13)$$

Threshold stress ( $q_t$ ) which can only be the maximum cyclic deviator stress possible within CSESS where cyclic stress equilibrium is inherent, becomes,

$$q_t = M p_i' \left( \frac{\kappa - \lambda}{\lambda} \right) \ln \left( \frac{p_i'}{p_o'} \right) \quad (14)$$

## 4 EXAMINATION AND VALIDATION OF MODEL

The above model for describing the level of threshold stress clearly states the fundamental importance of the volumetric compressibility index,  $\lambda$  and  $\kappa$ , in determining the threshold stress of clay.

Experimental result of the threshold stress for four series of undrained cyclic compression triaxial tests representing a wide range of stress history (i.e. normal-consolidated, over-consolidated ratio of 1.5, 4 and 20) on reconstituted saturated kaolinite clay, is shown in Figure 6. Comparison between the measured value of the threshold stress, and the predicted value using Equation (12) & (14) showed good agreement between the two (see Table 1). The discrepancy between the predicted and observed thresh-

old stress of the soil with an OCR = 20 was largely attributed to the anisotropic nature of the prepared clay sample, causing  $\Delta p' < 0$ , when the peak stress state intercepting CSESS at a slightly lower  $q_t$ .

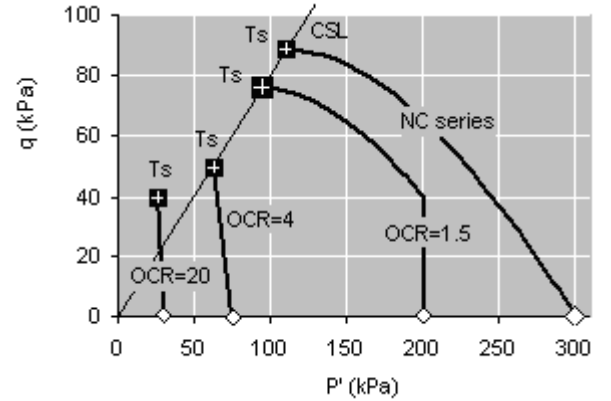


Figure 6 The line of cyclic stress equilibrium state (LCSES) for various consolidation states (kaolinite clay)

Table 1 Comparison of model-predicted values of threshold stress with the observed values from tests data (tests conducted on re-constituted kaolinite clay:  $M = 0.803$ ;  $\lambda = 0.176$ ;  $\kappa = 0.068$ )

Stress history	$p_i'$ (kPa)	$p_o'$ (kPa)	$q_t$ (kPa) (Predicted from Equation (12) & (14))	$q_t$ (kPa) (Observed)
Normal consolidated	300	300	88.6	90
OCR=1.5	200	300	75.8	75
OCR=4	75	300	51.9	50
OCR=20	30	600	44.3	40

## 5 CONCLUSION

Cyclic stress equilibrium state surface (CSESS) for undrained cyclic compression exists within the stress state boundary surface (SSBS). Developed from the original Cam-clay model, the proposed model described and defined the CSESS at constant volume, which derived the prediction equation for establishing the threshold stress of clay soil of non-sensitive nature under undrained cyclic compression. The proposed model enable the lower-bound threshold stress of otherwise unsaturated clay subgrade to be predicted without resort to lengthy testing programme. The model is relatively simple and input parameters can be easily obtained from basic laboratory tests. Measured data from the testing programme on a reconstituted clay confirmed the authenticity of the model. Modification and expansion of the proposed model for predicting threshold stress of soil in its un-

saturated state is possible. It is however recommended further tests on other clay type be conducted to substantiate the applicability of the model.

## REFERENCES:

- Ansal, A. M. (1989). "Undrained behavior of clay under cyclic shear stresses." *Journal of Geotechnical Engineering*, 115(7), 968-983.
- Carter, J. P., Booker, J. R., and Wroth, C. P. (1982). "A critical state soil model for cyclic loading." *Soil Mechanics - Transient and Cyclic Loads*, G. N. P. a. O. C. Zienkiewicz, ed., John Wiley & Sons Ltd.
- Egan, J. A., and Sangrey, D. A. "Critical state model for cyclic load pore pressure." *Earthquake engineering and soil dynamics: proceedings of the ASCE Geotechnical Engineering Division speciality conference*, 410-424.
- Heath, D. L., Shenton, M. J., Sparrow, R. W., and Waters, J. M. "Design of conventional rail track foundations." *Proceedings, Institute of Civil Engineers*, 251-267.
- Lefebvre, G. "Stability threshold for cyclic loading of saturated clay." *3rd Canadian Conference on Marine Geotechnical Engineering.*, St John's Newfoundl, Canada, 675-690.
- Procter, D. C. (1984). "Cyclic triaxial tests on remoulded clays." *Journal of Geotechnical Engineering*, 110(10), 1431-1445.
- Sangrey, D. A. (1969). "Effective stress response of a saturated clay soil to repeated loading." *Canadian Geotechnical Journal*, 6(3), 241-252.
- Schofield, A., and Wroth, P. (1968). *Critical state soil mechanics*, McGRAW - HILL Publishing Company Limited, London.
- Waters, J. M. (1968). "Track foundation design." *Railway Gazette*, 124(19), 734-737.
- Wood, D. M. (1990). *Soil behaviour and critical state soil mechanics*, Cambridge University Press.